

UB SAT 2009
Worksheet #11
SAT Functions



1. If $a \otimes b = \left(\frac{a}{b}\right)^2$, then $6 \otimes 2 =$

2. For all nonnegative numbers a , $\boxed{a} = \sqrt{a} + 1$. Which of the following is equal to the sum of $\boxed{36} + \boxed{64}$?

- (A) 10
- (B) 12
- (C) 16
- (D) 52
- (E) 102

3. For any number w , let $\#w\#$ be defined by the equation $\#w\# = -[w^2(w-1)]$.

What is the value of $\#-1\#$?

- (A) -2
- (B) -1
- (C) 0
- (D) 1
- (E) 2

4. Let $\ll x \gg = 2x - 1$ for all positive integers. If $\ll x \gg = 15$, then $x =$

- (A) 6
- (B) 7
- (C) 8
- (D) 15
- (E) 16

5. If $\triangle_a^b c = \frac{a+b}{c} + \frac{a+c}{b} + \frac{b+c}{a}$ for all nonzero a , b , and c , then $\triangle_2^1 3 =$

- (A) 12
- (B) 10
- (C) 8
- (D) 5
- (E) 3

Questions 6 and 7 refer to the following definition:

For all positive integers k ,

$$\text{let } \triangleleft k \triangleright = 2(k-1) \quad \text{if } k \text{ is even;}$$

$$\text{let } \triangleleft k \triangleright = \frac{1}{2}(k+1) \quad \text{if } k \text{ is odd.}$$

6. The product $\triangleleft 6 \triangleright \times \triangleleft 11 \triangleright =$

- (A) 25 (B) 30 (C) 31 (D) 59 (E) 60

7. If $\triangleleft N \triangleright$ is a multiple of 4, which of the following can be the value of N ?

- (A) 8 (B) 12 (C) 25 (D) 31 (E) 40

Questions 8 and 9 refer to the following definition:

Let the operation Φ be defined for all positive integers a and b by the equation

$$a\Phi b = ab - a$$

8. For what value of a is $a\Phi 4 = 24$?

- (A) 3 (B) 4 (C) 6 (D) $\frac{25}{4}$ (E) 8

9. Which of the following must be true?

- I. $a\Phi b = b\Phi a$
- II. $a\Phi(a+1) = a^2$
- III. $\frac{a}{2}\Phi\frac{b}{2} = \frac{a\Phi b}{2}$

- (A) I only
(B) II only
(C) III only
(D) I and III only
(E) II and III only

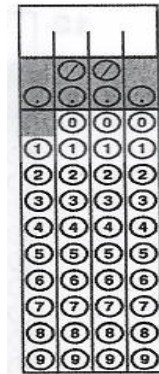
10. If $m \odot n$ is defined by the equation

$$m \odot n = \frac{m^2 - n + 1}{mn}, \text{ for all nonzero } m \text{ and } n,$$

then $3 \odot 1 =$

- (A) $\frac{9}{4}$
- (B) 3
- (C) $\frac{11}{3}$
- (D) 6
- (E) 9

11. Let $k \phi j$ be defined as the sum of all integers between k and j . For example, $5 \phi 9 = 6 + 7 + 8 = 21$. What is the value of $(80 \phi 110) - (81 \phi 109)$?



12. Let $\boxed{x} = \frac{x^2 + 1}{2}$ and $\bigcirc y = \frac{3y}{2}$, for all integers x and y . If $m = \bigcirc 2$, \boxed{m} is equal to which of the following?

- (A) $\frac{13}{8}$
- (B) $\frac{5}{2}$
- (C) $\frac{15}{4}$
- (D) 5
- (E) $\frac{37}{2}$

Questions 13 and 14 refer to the following definition:

$$\text{For all positive integers } y, \llbracket y \rrbracket = 2\sqrt{y}$$

13. Which of the following equals 8?

- (A) $\llbracket 4 \rrbracket$
- (B) $\llbracket 8 \rrbracket$
- (C) $\llbracket 16 \rrbracket$
- (D) $\llbracket 32 \rrbracket$
- (E) $\llbracket 64 \rrbracket$

14. $\llbracket y \rrbracket \times \llbracket y \rrbracket =$

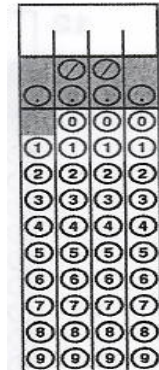
- (A) 2
- (B) 4
- (C) $4\sqrt{y}$
- (D) $4y$
- (E) $4y^2$

15. If $x \diamond y = (x - y)^2$ for all integers, which of the following must be true?

- I. $x \diamond y = y \diamond x$
 - II. $x \diamond y = x \diamond (-y)$
 - III. $x \diamond (-y) = (-x) \diamond y$
- (A) I only
 - (B) III only
 - (C) I and II
 - (D) I and III
 - (E) II and III

16. For all numbers x and y where $x + y \neq 0$, let

$$x \nabla y = \frac{x - y}{x + y}. \text{ If } 6 \nabla s = \frac{3}{4}, \text{ then } s =$$



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Answers

1. 9

2. C

3. E

4. C

5. C

6. E

7. D

8. E

9. B

10. B

11. 190

12. D

13. C

14. D

15. D

16. $\frac{6}{7}$